Mathematical models for cancer invasion

supervised by

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Description

Cancer cell migration is an essential stage in the development and expansion of tumours and their metastases. Once a malignant tumour arises in some part of the body, it can grow and make its way through the surrounding tissue matrix in order to reach the blood vessels. After transportation across the bloodstream and subsequent extravasation, tumour cells colonise distant sites where further neoplasms emerge. This process is known as metastasis.

Cancer is a very complicated illness. Construction of mathematical models describing growth and dispersion of cancer cells and their analytical and numerical study can contribute to a better understanding of the involved biological phenomena, enable predictions about the development and the extent of a tumour, and even suggest approaches to therapy improvements.

Evolution equations are among the most convenient and versatile modelling tools in this context. They allow to include different effects as 'building blocks' and can be treated analytically as well as numerically.

A reasonable minimal model for tumour invasion is obtained by coupling a reaction-diffusion-transport PDE for the density of the tumour with an ODE for the density of the surrounding tissue. Further components and various nonlinear effects could be introduced, such as: nonlinear diffusion, transport due to taxis (directed movement along the gradient of an attracting/repelling substance), nonlocality (in form of integral terms), modelling on different scales, etc. The resulting mixed systems of equations of different types (e.g., PDEs and ODEs) are highly nontrivial and not as yet well-studied. Often, new ideas are required in order to attack them analytically.

The goal of this project is thus to develop and investigate some systems for tumour invasion analytically. Performing numerical simulations, in order to illustrate the possible behaviour of solutions, would be welcome as an addition, but this is not mandatory.

Prerequisites

For this project solid knowledge of and inclination for mathematical analysis is indispensable. General acquaintance with PDEs is necessary. Even more important is to be familiar with basic tools from analysis in infinite dimensional spaces (compactness, weak convergence, linear and compact operators, etc.), as well as with integration theory (Lebesgue measure and integral).