Graded rings and toric methods in algebra

supervised by

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There is a fascinating and deep connection between partitions of $\mathbb{R}^n$ by cone-shaped subsets into "fans", and so-called toric varieties (geometric objects that can be described as solutions of polynomial equations of a rather simple type). Working with fans, toric varieties can be analysed combinatorially using diagram-theoretic methods, eschewing most of the complications of algebraic geometry. More importantly, the approach *via* diagrams of rings and modules makes it possible to study toric varieties in generalised contexts, for example, over *non-commutative* rings.

The basic notion underlying the theory is that of a LAURENT polynomial ring in several variables $L = R[x_1, x_1^{-1}, \cdots, x_n, x_n^{-1}]$, the collection of polynomials (with coefficients in the ring R) where negative and positive powers of the variables may occur. The combinatorial data associated with a toric variety can be used to construct other rings from L, and to inter-relate these rings in a meaningful manner.

One can often work in much greater generality: the LAURENT polynomial ring can be replaced by a *graded* ring, that is, a ring in which elements come equipped with a notion of "degree" similar to the usual notion of degree of a polynomial. The PhD project is about analysing this replacement step. Once the necessary machinery for graded rings has been set up many results of a formal nature carry over from the classical to the generalised setting without major problems. However, a closer look reveals surprising and subtle differences and complications, reflecting the more varied nature of graded rings as opposed to LAURENT polynomial rings.

The project may develop in various different directions, depending on the interests of the student. Possible topics include:

- an analysis of the algebraic K-theory of toric varieties in this generalised context (for example, it is expected that the K-theory of a projective generalised toric variety contains certain "trivial" summands);
- using toric methods to obtain results on tameness and finiteness conditions for chain complexes (for example, consider the "jump loci" for finite domination in the spirit of Sigma invariants from geometric group theory),
- extending results from affine toric algebra to the graded setting (for example, investigate how results connected with the "fundamental theorem for algebraic K-theory" transfer to the generalised setting).

The project is strongly algebraic in nature. The student wishing to undertake this kind of research must be willing to learn and work with the abstract mathematical machinery of graded algebra, homological algebra and algebraic K-theory.